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Will Your Factor Deliver? An Examination of Factor Robustness and Implementation Costs

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The multifactor investing framework has become very popular in the indexing community. Both academic and practitioner researchers have documented hundreds of equity factors. But which of these factors are likely to profit investors once implemented? We find that many of the documented factors lack robustness. Size and quality, two of the more prominent factors, show weak robustness, whereas value, momentum, illiquidity, and low beta are more robust. Further examining implementation characteristics, we find that liquidity-demanding factors, such as illiquidity and momentum, are associated with significantly higher trading costs than are other factors. Investors may be better off accessing these factors through active management rather than indexation.

A number of recent papers have documented the alarming trend of "300 + 40" in the investment industry. No, it is not the hedge fund industry upping its take to 300 bps in management fees plus 40% in performance fees, although that would certainly warrant concern. Rather, it is the proliferation of quantitative "factor strategies." According to estimates developed by Harvey, Liu, and Zhu (2016), there are at least 300 published factors, with roughly 40 newly discovered factors announced each year.[1] These "300 + 40" academic papers provide a valuable service to the investment community: They document an empirical pattern of excess returns of a strategy and explore the possible drivers of those returns. The drivers usually fall into one of two categories: (1) undiversifiable risk exposure (a risk-based explanation) or (2) the exploitation of mispricing that originates in market participants' psychological biases and limited arbitrage (a behavioral explanation). If a strategy is to persist in the future, it is important to know what may have caused it to persist in the past.

At the same time, few serious investors are likely to believe that all the 300-odd factor strategies would actually deliver reliable premiums in the future. Aside from a few egregious cases of research "mistakes" in which a claimed factor premium could not be replicated by other researchers, there are many other reasons to question the validity of the various exotic new sources of excess returns, which some academics mock as a "zoo of factors."[2] Skeptics argue that many of the documented factor premiums are the fruit of massive, intentional data mining. As early as the 1990s, when the number of discovered factors was much smaller, several scholars cautioned that many investment/factor strategies, whether billed as behavioral anomalies or otherwise, are a result of data mining (see, e.g., Lo and MacKinlay 1990; Black 1993; MacKinlay 1995). A kinder interpretation would simply acknowledge that if each of the thousands of professors, graduate students, and quantitative analysts were to backtest a single strategy every year, some would undoubtedly discover what appear to be winning strategies but are, in fact, lucky flukes.

Recently, some researchers have begun to explore the persistence of various factor premiums. McLean and Pontiff (2015) tested the out-of-sample performance of 97 equity factor strategies identified in the literature and found that 12 of them could not be replicated using similar data and time periods. In out-of-sample tests, they estimated that the reported factor premiums were inflated by an average of 26% because of data mining. Moreover, after a factor strategy becomes known, its premium falls by an average of 32% versus the published figure. Levi and Welch (2014) argued for shrinking the prospective factor premium to 30% of its historical value to account for estimation noise.

Given the large number of backtests conducted every year, a standard t-statistic of 2 is no longer a sufficient hurdle for establishing statistical significance. Bailey, Borwein, Lopez de Prado, and Zhu (2014, 2015); Harvey and Liu (2015); and Harvey

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et al. (2016) offered stricter statistical criteria for validating new factors. These new multiple-test statistical standards consider the number of backtests attempted, their degree of correlation, and publication bias. The statistical methods used seek to provide a summary measure that can help investors decide whether to accept or reject a factor. For example, Harvey et al. (2016) concluded that most of the recent factor discoveries are probably false, estimating that a *t*-statistic of 3 would be a more appropriate threshold for new factors today.

These theoretical and empirical explorations undoubtedly provide great insights into the market's attitude toward various risks and persistent investor behavioral biases as well as the probability of false discoveries. However, investors seeking to generate profits are more interested in the prospective excess returns, net of fees and expenses, that they might capture by investing in factor strategies. Although the practical matter of factor selection may seem straightforward, it is in fact quite difficult, especially for investors who lack the resources necessary to analyze the panoply of documented factors. For mainstream asset owners, a set of guidelines for identifying investable factor strategies with reliable premiums may be all that is needed.[3]

Because the primary aim of our study was to help investors navigate the multitude of factors, we adopted the heuristic approach developed by Hsu, Kalesnik, and Viswanathan (2015) for testing factor robustness. Using their framework, we report in this article a wide range of empirical evidence to help investors evaluate the popular factors that we analyzed. We also used the trading-cost model described in Aked and Moroz (2015) to estimate the return impact of turnover, thereby offering investors a more realistic estimation of the potential net-of-cost benefits of factor investing. Our net-of-cost analysis demonstrates the impact of implementation costs as a fraction of the estimated factor premium. This finding has implications for all factor strategies, especially for those in which the premium depends on relatively small or illiquid stocks. Finally, the factor analysis presented here also includes (as supplemental material at www.cfapubs.org/doi/suppl/10.2469/faj.v72.n5.6) "deep dives" into size and quality—factors that are commonly cited yet lack sufficient robustness in the global data.

Heuristic Guidelines

The heuristic approach to evaluating documented factors uses both qualitative and quantitative procedures, allowing investors to examine factor premiums from multiple viewpoints. The resulting multidimensional analysis is more informative than a test statistic, even one that attempts to partially adjust for the industry's collective data mining.

The Hsu–Kalesnik–Viswanathan analytical framework that we used is meant to be as intuitive as possible yet rigorous enough to serve its practical purpose. Tests of quality, the new darling in the factor zoo, vividly illustrate the problems with modern factor proliferation. Applying this methodology also reveals that even such long-standing factors as size can be invalidated as new knowledge comes to light. The following four points summarize the methodology that we used in our study:

Factors should be grounded in a long and deep academic literature. Taking advantage of academic research that is peer reviewed and generally free from undisclosed conflicts of interest is one of the best strategies for investors. A long literature debating the existence and persistence of a factor strategy, including rigorous attempts to debunk it, is critical to validating a factor. A factor strategy that does not attract follow-on research usually means that the factor has not survived academic scrutiny.

Factors should be robust across definitions. Publication bias occurs because results without significant *t*-statistics are almost never published. It is thus reasonable to view published factor definitions as overfitted to historical data, whether explicitly or implicitly—which, of course, overstates the forward excess return for factors. To obtain a more sensible *ex ante* estimate for a given factor premium, investors need to test a number of comparably reasonable strategies. For example, the average value premium computed from several similar value signals (e.g., the book-to-price, sales-to-price, trailing-earnings-to-price, dividend-to-price, and cash-flow-to-price ratios) should be a more representative estimate of the true value premium than one based on only the book-to-price ratio, which has the best in-sample performance. This perturbation approach—ascertaining the effect of small changes in the factor definition—can help investors identify factors that have been overfitted: the ones in which minor redefinitions tend to produce large variations in estimated premiums. For example, small changes in the definition of the quality factor can cause the estimated quality premium to go to zero or even become statistically negative.

Factors should be robust across geographies. Most research on factor investing is based on US data only. This tendency is partly driven by the availability of low-cost, high-quality US return and corporate financial data. Therefore, replicating a US-based study with non-US data may provide out-of-sample verification for a given factor strategy. Regardless of whether a factor premium discovered in the US data is driven by risk or by persistent investor behavior, the premium should show up in most countries/markets. It would be hard to explain why a risk exposure is priced only in the United States or a persistent behavioral bias occurs only in US investors.

Trading costs matter. Perhaps because academics are more interested in market efficiency and the underlying asset-pricing dynamics than in the real-world profitability of investment strategies, most academic research ignores implementation costs. To investors, however, costs are a performance drag that matters tremendously. The demand for low-cost products is undoubtedly a strong driver of investors' recent interest in the "smart beta" category of products—an indexing approach to capturing factor premiums.[4] However, some factor strategies and portfolio construction methodologies entail substantially higher turnover and/or transact in more illiquid securities. Adjusting simulated results for trading costs is a necessary step in meaningfully validating factor-based investment strategies.

In our study, we identified the factor-based investment strategies cited most often in the academic literature. We then used these factor definitions to build standard long–short portfolios, whose robustness we evaluated in two ways:

- We examined the *t*-statistics of the long–short portfolios' excess returns under various factor definitions and for different geographical regions. The *t*-statistic is related to the information ratio for each strategy when it is used to create over- and underweight positions in active long-only portfolios. This statistic is most relevant for investors who are sensitive to benchmark risk.
- We calculated the Sharpe ratios for the long and short portfolios separately. This procedure gives investors information on the bang-for-the-buck difference between investing in companies with positive exposures and investing in companies with negative exposures to a given factor. This statistic is most relevant for investors who can largely ignore benchmark risk and who are interested in the absolute risk–return characteristics of their investment.

In this article, we report the results, both gross and net of estimated transaction costs, from our robustness tests. We offer no hard-and-fast statistical thresholds for accepting or rejecting factors, but in most cases, the sensible conclusion seems uncontroversial. Because many innovations in smart-beta investing or factor strategies use long-only portfolios to access the target exposures, we provide separate statistics on the long and the short portfolios. We also demonstrate the size of the premium when the universe is restricted to large-cap stocks or small-cap stocks. Although we acknowledge that our research results do not map precisely to commercial products and strategies, we believe that our results offer relevant and useful guidance on forward-looking excess returns and potential implementation costs.

Evaluating Factor Robustness

Academia employs thousands of extremely well-educated financial economists whose debates and participation in the peer-review process improve our collective understanding of the financial markets. Thanks to their unremitting efforts, the probability of faulty research being published—let alone taking root and flourishing in the literature—is exceedingly remote. Although several researchers (Bailey et al. 2014, 2015; McLean and Pontiff 2015) have failed to replicate many factor strategies in-sample, interest in such strategies fortunately had waned long before researchers reminded us about them. Investors would be wise to recognize that "being published in a journal" is an insufficient qualification for an investment strategy.

To pare down the universe of factor strategies for examination, we first searched the Social Science Research Network (SSRN) database to identify factors that a significant number of research articles explored. We identified six popular equity factors that have at least 100 associated publications as determined by keywords in the title or abstract: illiquidity (570 SSRN hits), low beta (260 combined hits for low beta and low volatility), value (2,327 hits), momentum (457 hits), size (1,167 hits), and quality (1,700 combined hits for profitability, distress, accruals, and quality).[5] Many factors did not make it through this filter. For example, a factor strategy based on short-sale restrictions had only 26 hits, and an IPO factor had only 86 hits. Note that we used Harvey et al. (2016) for our *initial* universe of factor exploration.[6]

For our analysis, we formed portfolios on the basis of factor characteristics to measure the historical unadjusted and risk-adjusted return advantage of each factor. We first divided the universe into large and small stocks. Following standard practice in the academic literature, we examined factors in the subuniverses of large and small stocks separately. We studied the size factor separately for three main reasons: (1) We wanted our results to be comparable to what is reported in the literature; (2) size seems to be associated with a common economic driver, resulting in stronger effects among small stocks for several factors; and (3) factors may have more pronounced effects among smaller stocks owing to arguably lower investor interest as well as lower liquidity. Following Fama and French (1993), we defined large (small) stocks in the US market as those whose prior-month market capitalizations are above (below) the median market cap on the NYSE. Following Fama and French (2012), we defined large (small) stocks in international markets as those in the top 90% (bottom

10%) by cumulative market cap. Because the subuniverse of large stocks accounted for approximately 90% of total market capitalization, the estimated large-stock factor returns are closely indicative of factor performance in the entire universe.

We then partitioned the large and small subuniverses by factor strategy—value, momentum, low beta, quality, or illiquidity (see Appendix A)—to construct high-characteristic and low-characteristic portfolios. For example, within the US large-stock subuniverse, we constructed the value stock portfolio with stocks above the 70th percentile on the NYSE by book-to-market ratio; we constructed the growth stock portfolio with the bottom 30% by the same measure. We then weighted stocks by market capitalization within each of the four resulting size-characteristic portfolios (e.g., for the value factor: large value, large growth, small value, small growth). This methodology is similar to that of Fama and French (1993) but with a few key differences: (1) data cleaning and lagging (Fama and French lagged market price data by six months, whereas we did not because that information was available immediately), (2) the rebalancing month (January versus July), and (3) dependent versus independent sorting (Fama and French sorted by size and characteristic independently, whereas we performed a sequential sort to ensure that portfolios were adequately populated and transaction costs could be fairly compared across factors). In our study, we grouped stocks into the following regions: United States, Japan, United Kingdom, Europe ex-UK, and Global. In assessing robustness across geographical regions, we used the most standard definition for a given factor to save space (see Appendix A for the definitions). Only for the quality factor, which lacks a standard definition, do we provide definitional variations across regions.

In addition to examining the performance of the various factor strategies in the large- and small-stock subuniverses separately, we examined the performance of the "combined" portfolio—50% invested in the large-characteristic portfolio and 50% in the small-characteristic portfolio. We rebalanced portfolios annually each January, with the exception of momentum, which we rebalanced monthly. Unless otherwise noted, US data cover 1967–2014 and international data cover 1987–2014. We used returns (calculated as geometrically annualized averages of monthly returns), volatilities, and Sharpe ratios of the resulting portfolios, along with statistical tests of differences in returns and Sharpe ratios, to determine whether each factor provides improved risk and/or return characteristics. We determined the significance of differences in returns with a *t*-test of the average monthly returns of the associated long–short portfolio. Using the bootstrapping method, we ascertained the significance of differences in Sharpe ratios: We sampled monthly returns (with replacement; i.e., any observation may be sampled more than once) of high- and low-characteristic portfolios and coinciding risk-free rates to create a bootstrapped distribution of Sharpe ratios for hypothesis testing. We used the significance test statistics to determine whether each factor provides improved risk and/or return characteristics.

Our evaluations of factor robustness are presented in roughly the same order in which the factors were first documented.

Low-Beta Factor. Haugen and Heins (1975), perhaps as a byproduct of the empirical testing of the CAPM, documented that stocks with a higher beta than the equity market portfolio do not produce higher returns.[7] They found that low-beta stocks, on average, perform on par with or often better than high-beta stocks.

A number of rational and behavioral reasons may help explain the low-beta anomaly and its persistence. Much of the literature has focused on the low Sharpe ratio for high-beta stocks driven by excess demand: (1) Investors with leverage constraints or leverage aversion may use high-beta stocks to increase portfolio returns;[8] (2) investors may use high-beta stocks, which tend to have a large positive upside (positive skew), to speculate;[9] and (3) sell-side analysts tend to substantially inflate growth forecasts for high-beta companies, generating investor optimism, which in turn generates short-term fund flows into their equity shares.[10] However, the low-beta anomaly is difficult for institutional investors to exploit. Underweighting high-beta stocks simply generates too much tracking error in a traditional long-only portfolio.[11] As a poor information ratio trading signal, the low-beta strategy has been more of a research curiosity than a useful investment strategy.

For empirical tests of the low-beta factor, we used the Frazzini and Pedersen (2014) estimation of beta as our primary definition.[12] **Table 1** presents returns and Sharpe ratios for low-beta and high-beta portfolios, along with *t*-statistics of both the differences in returns (long-minus-short portfolio) and the differences in the long and short portfolios' Sharpe ratios. Panel A of Table 1 shows portfolio returns for small perturbations in the strategy definition. In addition to our primary definition, we used prior-one-year volatility, prior-three-year beta, and prior-three-year volatility estimated with daily data. As Panel A shows uniformly, low-beta (low-volatility) stocks have small improvements in returns over high-beta (high-volatility) stocks, accompanied by significant reductions in risk.

With respect to *t*-statistics, the differences in returns between low-volatility and high-volatility stocks are not statistically significant, even though

Table 1. Robustness of the Low-Beta Factor, 1967–2014 (US Data) and 1987–2014 (International Data)

Definition	Low Beta Return	Low Beta Volatility	High Beta Return	High Beta Volatility	t-Statistic of Long – Short
A. Robustness of low-beta factor across definitions: Returns					
Large					
Low beta	11.4%	12.1%	8.0%	20.8%	0.79
Low volatility	10.8	12.4	7.8	23.5	0.31
Low beta, 3 years	11.5	12.2	8.3	19.5	0.86
Low volatility, 3 years	11.0	12.7	8.5	23.0	0.18
Small					
Low beta	15.5	15.0	10.4	28.4	0.67
Low volatility	15.3	14.9	8.4	28.4	1.26
Low beta, 3 years	14.9	15.4	10.5	27.7	0.55
Low volatility, 3 years	15.1	15.1	10.1	27.6	0.72
Combined					
Low beta	13.6	12.8	9.5	23.7	0.77
Low volatility	13.1	13.0	8.3	25.2	0.84
Low beta, 3 years	13.3	13.0	9.7	22.8	0.75
Low volatility, 3 years	13.2	13.1	9.5	24.6	0.48

Definition	Sharpe Ratio Low Beta	Sharpe Ratio High Beta	t-Statistic of Sharpe Ratio Difference	Significant
B. Robustness of low-beta factor across definitions: Sharpe ratios				
Large				
Low beta	0.53	0.14	3.26*	Yes
Low volatility	0.46	0.11	3.09*	Yes
Low beta, 3 years	0.53	0.17	3.07*	Yes
Low volatility, 3 years	0.47	0.15	2.86*	Yes
Small				Yes
Low beta	0.69	0.19	6.06*	Yes
Low volatility	0.68	0.12	6.01*	Yes
Low beta, 3 years	0.64	0.20	5.57*	Yes
Low volatility, 3 years	0.67	0.18	5.27*	Yes
Combined				
Low beta	0.66	0.19	5.11*	Yes
Low volatility	0.62	0.13	5.04*	Yes
Low beta, 3 years	0.63	0.20	4.75*	Yes
Low volatility, 3 years	0.62	0.18	4.49*	Yes

Region	Low Beta Return	Low Beta Volatility	High Beta Return	High Beta Volatility	t-Statistic of Long – Short
C. Robustness of low-beta factor across geographical markets: Returns					
Large					
United States	11.4%	12.1%	8.0%	20.8%	0.79
United Kingdom	9.9	16.3	6.0	22.1	0.87
Europe ex-UK	10.4	14.5	6.3	23.2	0.84
Japan	6.0	17.7	–2.4	26.4	1.88
Global	9.4	11.7	6.5	20.4	0.51
Small					
United States	15.5	15.0	10.4	28.4	0.67
United Kingdom	10.7	16.5	7.8	22.1	0.76
Europe ex-UK	12.2	14.5	8.4	23.0	0.70
Japan	6.2	21.4	4.1	31.2	–0.16
Global	11.8	11.9	9.0	21.0	0.45
Combined					
United States	13.6	12.8	9.5	23.7	0.77
United Kingdom	10.5	15.4	7.1	21.3	0.95
Europe ex-UK	11.4	14.0	7.5	22.5	0.83
Japan	6.3	18.6	1.2	27.6	0.99
Global	10.7	11.3	7.9	20.1	0.50

(continued)

Table 1. Robustness of the Low-Beta Factor, 1967–2014 (US Data) and 1987–2014 (International Data) (continued)

Region	Sharpe Ratio: Low Beta	Sharpe Ratio: High Beta	t-Statistic of Sharpe Ratio Difference	Significant
D. Robustness of low-beta factor across geographical markets: Sharpe ratios				
Large				
United States	0.53	0.14	3.26*	Yes
United Kingdom	0.39	0.11	1.89	No
Europe ex-UK	0.48	0.12	3.01*	Yes
Japan	0.15	–0.22	2.70*	Yes
Global	0.51	0.15	2.65*	Yes
Small				
United States	0.69	0.19	6.06*	Yes
United Kingdom	0.44	0.20	2.27*	Yes
Europe ex-UK	0.61	0.22	3.17*	Yes
Japan	0.13	0.02	1.07	No
Global	0.70	0.26	3.87*	Yes
Combined				
United States	0.66	0.19	5.11*	Yes
United Kingdom	0.46	0.17	2.65*	Yes
Europe ex-UK	0.57	0.18	3.40*	Yes
Japan	0.15	–0.08	2.15*	Yes
Global	0.64	0.22	3.49*	Yes

*Significant at the 5% level.

Sources: Research Affiliates, LLC, using CRSP/Compustat and Worldscope/Datastream data.

they look economically large. The *t*-statistics of the long–short portfolios are all below 2 because the long–short portfolio is extremely volatile owing to its large negative equity market beta exposure. In other words, the low-beta factor tilt usually introduces very large tracking errors into a portfolio, making the excess return unattractive as measured by its information ratio—which suggests that the low-beta strategy is potentially inappropriate for investors who are averse to deviating from a benchmark.

For investors who can overlook the tracking error issue, the Sharpe ratio comparisons presented in Panel B of Table 1 should prove far more illuminating. Low-beta stock portfolios consistently have more attractive Sharpe ratios than high-beta stock portfolios.

As a technical aside, we note that Frazzini and Pedersen (2014) constructed a "beta neutral" low-beta factor, BAB (betting against beta), which is statistically better behaved than the standard long–short low-beta portfolio.[13] We do not report the BAB calculation, however, because it does not correspond to feasible over- and underweights for the traditional long-only portfolios that are relevant to most asset owners.

Panels C and D of Table 1 report the robustness of the low-beta factor across geographical markets. As before, we observe significantly improved risk–return characteristics associated with low-beta stocks. With respect to *t*-statistics, the only two instances in which the differences in Sharpe ratios are not statistically significant are the UK large-stock and Japanese small-stock universes. In both cases, however, the low-beta stocks still provide economically significant risk/reward improvements.

Given that low-beta stocks consistently provide risk–return advantages that are both economically and statistically significant across regions regardless of perturbations in definition, we conclude that the low-beta factor strategy has been a robust source of excess performance for investors who can take on the requisite tracking error. Because many institutions continue to shun this strategy for its high tracking error, the low-beta anomaly should continue to persist.[14]

Value Factor. Value as a factor strategy can be traced to Sanjoy Basu (1977), who used the price-to-earnings characteristic to select stocks. Jacobs and Levy (1988) found that different definitions of value as expressed by various ratios of company accounting fundamentals to stock prices (e.g., book-to-price and dividend-to-price ratios) capture largely the same anomaly. The value premium may be attributable to risk and/or behavioral bias.

Fama and French (1993) showed that value stocks move together as if responding to a common macro shock. This observation set in motion the development of several models, including those of Campbell and Vuolteenaho (2004) and Zhang (2005), who argued that capital-intensive companies with more irreversible investments (as proxied by high book-to-price ratios) are more exposed to shocks to the economy.

Lakonishok, Shleifer, and Vishny (1994) showed that the co-movement does not seem to be driven by a priced risk. Their finding led to models suggested by Chan and Lakonishok (2004) and Barber and Odean (2008) whereby low-book-to-price stocks are not so much "growth" as overvalued owing to the glitz and hype fueled by conflicted Wall Street analysts and the popular financial media.

The standard definition of value uses the book-to-price ratio. We also included other definitions: trailing earnings to price, trailing cash flows to price, and trailing dividends to price. The results are displayed in Panels A and B of **Table 2**. For all definitions, we see economically significant differences in returns between value and growth stocks. Note that at first glance, high-dividend-to-price stocks do not appear to meaningfully outperform low-dividend-to-price stocks. However, this observation is more an indictment of the standard statistical methodology applied than a statement about the efficacy of the dividend strategy. High-dividend-yielding stocks are simply much less risky (lower volatility, as shown in Panel A) than low-dividend-yielding stocks, and thus the difference portfolio—going long in high-dividend and short in low-dividend stocks—is quite volatile, just as it was for the low-beta factor. One can interpret this result to mean that dividend yield, like low beta, is a low-information-ratio strategy for creating overweights and underweights in a long-only active portfolio.

Looking at the Sharpe ratios in Panel B, however, we see that all definitions of value provide statistically better risk-adjusted returns. The value characteristic defined by high dividend yield is the most effective factor strategy by Sharpe ratio.

The international evidence reported in Panels C and D of Table 2 shows a similarly robust pattern of value outperforming growth. The only statistically insignificant outcome is the value portfolio in the UK large-stock universe. (Outliers are inevitable in any honest empirical study.) Our results are consistent with Asness, Moskowitz, and Pedersen (2013), who also documented the value effect internationally and found that value outperforms in nonequity asset classes.

Size Factor. Using US equity data, Rolf Banz (1981) documented that stocks with small market capitalizations tend to outperform stocks with large market capitalizations. What might account for the small-cap premium? Several explanations have been offered: (1) Small stocks expose investors to some undiversifiable risk—potentially credit shocks—because small companies are more capital constrained (see Fama and French 1993); (2) Shumway (1997) and Shumway and Warther (1999) found that the small-cap premium may be driven by data mistakes caused by the improper treatment of the delisting returns of stocks; and (3) Arnott, Hsu, Liu, and Markowitz (2015), using a noise-in-price model, argued that small-cap companies are more likely to be cheap, thus offering superior long-term returns. That same model, however, predicts that the small-cap premium would decay to zero over time. The second and third explanations should certainly alert investors to examine carefully the evidence for the existence and reliability of the small-cap premium.

Table 3 reports the robustness of the size factor. Panel A shows how the size factor responds to variations in definition. It is standard in the academic literature to use the 50th percentile on the NYSE to separate large and small companies. In our study, we varied the cutoff points to include the 75th and 25th percentiles as well. On average, small-cap stocks do provide higher returns than large-cap stocks, as reported in Panel A. Taking into account the excess volatility risk associated with small-cap stocks, however, the Sharpe ratios in Panel B show that no definition of small-stock portfolios delivers statistically significant risk-adjusted return benefits. Note that small-cap portfolios generally also exhibit a value bias: When we further adjusted the small-cap excess return for the value effect, the size premium fell close to zero (not shown here). As we see in Panels C and D of Table 3, no portfolio (with the exception of the US portfolio mentioned earlier) exhibits a statistically significant return advantage, whether risk adjusted or not.

At first glance, the size premium lacks robustness—an extremely surprising observation given that size is one of the best-established and most widely cited factors. For a closer examination of the size factor, see Appendix B (posted as supplemental material at www.cfapubs.org/doi/suppl/10.2469/faj.v72.n5.6), which describes our test of a longer sample covering more regions to provide readers with more data points to reach their own conclusions.

Momentum Factor. Momentum as a factor strategy originated with Jegadeesh and Titman (1993), who first documented that stocks that have recently outperformed (underperformed) continue to outperform (underperform). Building on this observation, Carhart (1997) defined and tested a long–short factor that became part of the standard Fama–French–Carhart four-factor pricing model. Jegadeesh and Titman attributed the momentum effect to investors' systematic underreaction to

Table 2. Robustness of the Value Factor, 1967–2014 (US Data) and 1987–2014 (International Data)

	Value		Growth		t-Statistic of
Definition	Return	Volatility	Return	Volatility	Long – Short
A. Robustness of value factor across definitions: Returns					
Large					
Book to price	13.1%	16.7%	9.3%	16.8%	2.02*
Earnings to price	13.3	16.0	8.8	17.8	2.14*
Cash flow to price	13.0	16.3	9.2	17.3	1.92
Dividends to price	12.7	13.9	9.4	20.0	0.89
Small					
Book to price	16.6	23.2	10.5	22.8	3.04*
Earnings to price	15.9	20.7	10.2	25.3	2.11*
Cash flow to price	17.0	22.5	10.2	23.1	3.17*
Dividends to price	15.4	16.7	11.2	25.1	0.96
Combined					
Book to price	15.0	19.2	10.1	18.9	2.77*
Earnings to price	14.8	17.6	9.8	20.6	2.32*
Cash flow to price	15.2	18.7	9.9	19.3	2.80*
Dividends to price	14.2	14.7	10.5	21.7	0.99

	Sharpe Ratio			
Definition	Value	Growth	t-Statistic of Sharpe Ratio Difference	Significant
B. Robustness of value factor across definitions: Sharpe ratios				
Large				
Book to price	0.48	0.25	2.11*	Yes
Earnings to price	0.52	0.21	2.80*	Yes
Cash flow to price	0.48	0.24	2.38*	Yes
Dividends to price	0.55	0.22	2.66*	Yes
Small				
Book to price	0.50	0.24	3.58*	Yes
Earnings to price	0.53	0.20	4.31*	Yes
Cash flow to price	0.53	0.22	3.94*	Yes
Dividends to price	0.62	0.24	4.36*	Yes
Combined				
Book to price	0.52	0.27	2.99*	Yes
Earnings to price	0.55	0.23	3.76*	Yes
Cash flow to price	0.54	0.25	3.39*	Yes
Dividends to price	0.62	0.25	3.70*	Yes

	Value		Growth		t-Statistic of
Region	Return	Volatility	Return	Volatility	Long – Short
C. Robustness of value factor across geographical markets: Returns					
Large					
United States	13.1%	16.7%	9.3%	16.8%	2.02*
United Kingdom	10.1	20.8	9.4	16.8	0.57
Europe ex-UK	11.2	20.9	5.1	18.2	2.68*
Japan	8.0	22.0	–2.5	23.6	3.30*
Global	11.2	16.6	5.6	16.6	2.52*
Small					
United States	16.6	23.2	10.5	22.8	3.04*
United Kingdom	13.5	20.6	6.7	21.1	2.89*
Europe ex-UK	13.8	19.9	6.5	18.9	3.56*
Japan	10.5	26.6	1.0	25.1	4.54*
Global	13.8	18.0	6.0	18.1	3.81*

(continued)

Table 2. Robustness of the Value Factor, 1967–2014 (US Data) and 1987–2014 (International Data) (continued)

Definition	Value Return	Value Volatility	Growth Return	Growth Volatility	*t*-Statistic of Long – Short
Combined					
United States	15.0	19.2	10.1	18.9	2.77*
United Kingdom	12.0	19.9	8.3	17.9	1.88
Europe ex-UK	12.6	19.9	5.9	18.0	3.34*
Japan	9.5	23.6	–0.4	23.0	4.33*
Global	12.6	16.9	6.0	16.6	3.38*

Region	Sharpe Ratio Value	Sharpe Ratio Growth	*t*-Statistic of Sharpe Ratio Difference	Significant
D. Robustness of value factor across geographical markets: Sharpe ratios				
Large				
United States	0.48	0.25	2.11*	Yes
United Kingdom	0.32	0.35	–0.22	No
Europe ex-UK	0.37	0.09	2.41*	Yes
Japan	0.21	–0.25	3.49*	Yes
Global	0.46	0.13	2.52*	Yes
Small				
United States	0.50	0.24	3.58*	Yes
United Kingdom	0.49	0.15	3.23*	Yes
Europe ex-UK	0.52	0.16	3.49*	Yes
Japan	0.27	–0.10	4.64*	Yes
Global	0.58	0.14	4.18*	Yes
Combined				
United States	0.52	0.27	2.99*	Yes
United Kingdom	0.43	0.27	1.45	No
Europe ex-UK	0.46	0.14	3.14*	Yes
Japan	0.26	–0.17	4.41*	Yes
Global	0.54	0.15	3.43*	Yes

*Significant at the 5% level.

Sources: Research Affiliates, LLC, using CRSP/Compustat and Worldscope/Datastream data.

positive and negative news; because their attention is limited, investors simply do not notice or react appropriately to relevant information.[15] Asness (1994) and Barberis, Shleifer, and Vishny (1998) noted that underreaction can, in due course, give way to herding as investors pile into winner stocks. This eventual overreaction can lead to long-horizon price mean reversion, giving rise to the value effect. Daniel and Moskowitz (2013) showed that momentum strategies experience crashes from time to time; this feature, combined with the high turnover and potential transaction costs,[16] probably contributes to the persistence of the phenomenon.

The typical momentum strategy looks at the past year of returns, skipping the most recent month to adjust for short-horizon mean reversion.[17] The holding period is usually one month;[18] because the momentum signal was shorter lived than the others in our study, we rebalanced it more frequently. Panels A and B of **Table 4** show variations in the definition of momentum, with both the formation (look-back) period and the holding period modified. We can see that the momentum strategy is far more reliable in the small-cap subuniverse. In the large-cap subuniverse, the strategy often does not produce a statistically positive advantage because the definition of the momentum strategy varies; this lack of robustness holds up whether measured by the Sharpe ratio or the information ratio.

Panels C and D of Table 4 consider the efficacy of the momentum factor strategy in various regions. We observe again that momentum is far stronger in the small-cap subuniverse. The exception is Japan, well known as a market where the value premium is very strong but the momentum premium is nonexistent. Note that the standard definition of momentum is largely effective in the large-cap domain in other regions. As with all empirical analysis, there is no

Table 3. Robustness of the Size Factor, 1967–2014 (US Data) and 1987–2014 (International Data)

Definition	Small Return	Small Volatility	Big Return	Big Volatility	*t*-Statistic of Long – Short
A. Robustness of size factor across definitions: Returns					
50% Small, 50% big	12.7%	20.7%	10.3%	15.3%	1.97*
75% Small, 25% big	12.5	19.1	10.1	15.1	2.21*
25% Small, 75% big	12.9	22.0	10.5	15.5	1.83

Definition	Sharpe Ratio Small	Sharpe Ratio Big	*t*-Statistic of Sharpe Ratio Difference	Significant
B. Robustness of size factor across definitions: Sharpe ratios				
50% Small, 50% big	0.37	0.34	0.29	No
75% Small, 25% big	0.39	0.33	0.78	No
25% Small, 75% big	0.35	0.35	0.05	No

Region	Small Return	Small Volatility	Big Return	Big Volatility	*t*-Statistic of Long – Short
C. Robustness of size factor across geographical markets: Returns					
United States	12.7%	20.7%	10.3%	15.3%	1.97*
United Kingdom	9.7	19.4	9.2	17.6	0.45
Europe ex-UK	10.5	18.0	8.6	18.5	1.02
Japan	5.8	24.4	1.8	21.6	1.83
Global	10.5	16.5	8.1	15.7	1.48

Region	Sharpe Ratio Small	Sharpe Ratio Big	*t*-Statistic of Sharpe Ratio Difference	Significant
D. Robustness of size factor across geographical markets: Sharpe ratios				
United States	0.37	0.34	0.29	No
United Kingdom	0.32	0.33	–0.05	No
Europe ex-UK	0.39	0.28	1.29	No
Japan	0.10	–0.07	1.59	No
Global	0.42	0.30	1.37	No

*Significant at the 5% level.

Sources: Research Affiliates, LLC, using CRSP/Compustat and Worldscope/Datastream data.

hard-and-fast rule by which to declare the momentum premium meaningful or not in the large-cap application. Reasonable people can disagree, especially when considering transaction costs.

Illiquidity Factor. Amihud (2002) and Pástor and Stambaugh (2003), among others, have shown that investors are compensated for holding illiquid stocks. The theoretical rationale is intuitive. Investors demand a risk premium for holding illiquid securities, which are hard to trade and experience extreme price losses in crises.

There are multiple ways to measure the illiquidity risk associated with a stock. In our study, we defined a stock as illiquid by its average adjusted daily volume (ADV) over the last month, which is a standard measurement of liquidity for equity traders.[19] Panels A and B of **Table 5** present additional definitions that include ADV over 6 and 12 months. In all cases, the portfolios of illiquid stocks outperform the more liquid ones, with an economically significant difference in returns. With respect to Sharpe ratios, we observe a uniform and significant risk-adjusted return benefit from holding illiquid stocks.

Looking at the international data in Panels C and D of Table 5, however, we observe weak evidence for the illiquidity premium. In all cases in the large-cap subuniverse, the Sharpe ratio for liquid stocks is weaker than that for illiquid stocks, although the difference is not statistically significant outside the United States. In the subuniverse of small companies, evidence for the illiquidity premium is even weaker.

On the basis of the statistics presented here, we conclude that there is mixed evidence in favor of an illiquidity premium. In the US market, the illiquidity premium seems to be strong and robust; internationally, illiquidity as defined by ADV does not seem to offer a premium. However, given that Amihud, Hameed, Kang, and Zhang (2015) have demonstrated

Table 4. Robustness of the Momentum Factor, 1967–2014 (US Data) and 1987–2014 (International Data)

Definition	Winners Return	Winners Volatility	Losers Return	Losers Volatility	t-Statistic of Long – Short
A. Robustness of momentum factor across definitions: Returns					
Large					
–2 to –12 Months	13.0%	17.2%	8.3%	18.7%	1.89
–2 to –12 Months, 3-month hold	12.3	17.5	8.3	18.5	1.67
–2 to –12 Months, 1-year hold	11.2	17.5	9.3	17.5	0.92
–2 to –6 Months	10.4	16.9	10.7	18.8	–0.29
–1 to –12 Months	12.4	17.0	9.3	19.3	1.11
Small					
–2 to –12 Months	17.9	21.2	3.7	27.1	4.99*
–2 to –12 Months, 3-month hold	16.3	21.3	4.3	26.4	4.51*
–2 to –12 Months, 1-year hold	14.7	21.2	8.4	25.1	2.69*
–2 to –6 Months	15.3	21.2	5.6	26.7	3.54*
–1 to –12 Months	16.5	20.9	5.8	27.9	3.24*
Combined					
–2 to –12 Months	15.6	18.5	6.3	22.0	3.74*
–2 to –12 Months, 3-month hold	14.5	18.7	6.6	21.6	3.36*
–2 to –12 Months, 1-year hold	13.1	18.5	9.1	20.5	1.95
–2 to –6 Months	13.0	18.3	8.4	21.9	1.83
–1 to –12 Months	14.6	18.2	7.9	22.7	2.39*

Definition	Sharpe Ratio Winners	Sharpe Ratio Losers	t-Statistic of Sharpe Ratio Difference	Significant
B. Robustness of momentum factor across definitions: Sharpe ratios				
Large				
–2 to –12 Months	0.46	0.17	2.27*	Yes
–2 to –12 Months, 3-month hold	0.41	0.18	1.91	No
–2 to –12 Months, 1-year hold	0.35	0.24	0.97	No
–2 to –6 Months	0.32	0.30	0.14	No
–1 to –12 Months	0.43	0.22	1.64	No
Small				
–2 to –12 Months	0.61	–0.05	6.30*	Yes
–2 to –12 Months, 3-month hold	0.53	–0.03	5.74*	Yes
–2 to –12 Months, 1-year hold	0.45	0.13	4.27*	Yes
–2 to –6 Months	0.49	0.02	4.94*	Yes
–1 to –12 Months	0.55	0.03	5.02*	Yes
Combined				
–2 to –12 Months	0.57	0.05	4.65*	Yes
–2 to –12 Months, 3-month hold	0.50	0.07	4.13*	Yes
–2 to –12 Months, 1-year hold	0.43	0.20	2.71*	Yes
–2 to –6 Months	0.43	0.15	2.86*	Yes
–1 to –12 Months	0.52	0.12	3.59*	Yes

Region	Winners Return	Winners Volatility	Losers Return	Losers Volatility	t-Statistic of Long – Short
C. Robustness of momentum factor across geographical markets: Returns					
Large					
United States	13.0%	17.2%	8.3%	18.7%	1.89
United Kingdom	12.1	19.3	2.9	23.3	2.37*
Europe ex-UK	11.1	18.5	4.4	23.0	1.71
Japan	2.7	22.9	0.8	25.5	0.34
Global	10.3	16.8	4.9	19.3	1.60

(continued)

Table 4. Robustness of the Momentum Factor, 1967–2014 (US Data) and 1987–2014 (International Data) (continued)

	Winners		Losers		t-Statistic of
Region	Return	Volatility	Return	Volatility	Long – Short
Small					
United States	17.9	21.2	3.7	27.1	4.99*
United Kingdom	16.6	19.9	–5.6	27.1	5.92*
Europe ex-UK	17.3	17.5	–2.2	24.5	5.57*
Japan	5.6	23.7	2.6	29.0	0.53
Global	14.4	16.7	0.0	22.2	4.81*
Combined					
United States	15.6	18.5	6.3	22.0	3.74*
United Kingdom	14.5	18.7	–1.2	24.1	4.50*
Europe ex-UK	14.3	17.3	1.2	23.2	3.87*
Japan	4.4	22.1	2.0	26.2	0.46
Global	12.5	16.3	2.6	20.1	3.31*

	Sharpe Ratio		t-Statistic of Sharpe	
Region	Winners	Losers	Ratio Difference	Significant
D. Robustness of momentum factor across geographical markets: Sharpe ratios				
Large				
United States	0.46	0.17	2.27*	Yes
United Kingdom	0.45	–0.03	2.88*	Yes
Europe ex-UK	0.42	0.04	2.54*	Yes
Japan	–0.03	–0.10	0.47	No
Global	0.41	0.07	2.01*	Yes
Small				
United States	0.61	–0.05	6.30*	Yes
United Kingdom	0.66	–0.33	6.33*	Yes
Europe ex-UK	0.79	–0.23	6.44*	Yes
Japan	0.09	–0.03	1.14	No
Global	0.66	–0.15	5.26*	Yes
Combined				
United States	0.57	0.05	4.65*	Yes
United Kingdom	0.59	–0.19	5.08*	Yes
Europe ex-UK	0.63	–0.10	4.96*	Yes
Japan	0.04	–0.06	0.81	No
Global	0.55	–0.04	3.91*	Yes

*Significant at the 5% level.

Sources: Research Affiliates, LLC, using CRSP/Compustat and Worldscope/Datastream data.

international persistence of the illiquidity measure under a substantially more complex definition of illiquidity, we are not ready to rule out the existence of an illiquidity premium. Even if there is a strong and persistent illiquidity premium, a question naturally arises: How much of the factor's return advantage can be translated into after-cost returns for investors? We discuss this question later in the article.

Quality Factor. Superficially, an investment strategy that aims to capture a quality factor premium implies buying "high-quality" companies and avoiding "low-quality" companies. The problem is how to define "quality" more precisely. The following are a few of the many quality-related measures that have been studied in the academic literature:

- *Profitability:* Novy-Marx (2013) defined quality in terms of the gross-profits-to-assets ratio; Fama and French (2015) introduced operating profit as a measure of profitability in their five-factor model.
- *Accruals:* Sloan (1996) and Hirshleifer, Hou, Teoh, and Zhang (2004) introduced accruals-related measures to signal potential problems with accounting practices.
- *Advertising and R&D expenses:* Chauvin and Hirschey (1993) studied advertising and R&D expenses and their effects on equity returns.

Table 5. Robustness of the Illiquidity Factor, 1967–2014 (US Data) and 1987–2014 (International Data)

Definition	Illiquid Return	Illiquid Volatility	Liquid Return	Liquid Volatility	t-Statistic of Long – Short
A. Robustness of illiquidity factor across definitions: Returns					
Large					
1-Month ADV	12.8%	15.7%	9.7%	15.4%	2.43*
6-Month ADV	13.3	15.3	9.7	15.4	2.80*
12-Month ADV	13.1	15.1	9.8	15.4	2.58*
Small					
1-Month ADV	15.8	18.7	11.0	23.9	2.08*
6-Month ADV	16.2	18.7	10.9	24.0	2.37*
12-Month ADV	16.6	18.4	11.0	24.1	2.43*
Combined					
1-Month ADV	14.4	16.6	10.6	18.9	2.79*
6-Month ADV	14.9	16.4	10.6	18.9	3.20*
12-Month ADV	15.0	16.2	10.7	18.9	3.15*

Definition	Sharpe Ratio Illiquid	Sharpe Ratio Liquid	t-Statistic of Sharpe Ratio Difference	Significant
B. Robustness of illiquidity factor across definitions: Sharpe ratios				
Large				
1-Month ADV	0.49	0.30	2.39*	Yes
6-Month ADV	0.54	0.30	2.91*	Yes
12-Month ADV	0.53	0.31	2.78*	Yes
Small				
1-Month ADV	0.57	0.25	4.89*	Yes
6-Month ADV	0.60	0.24	5.45*	Yes
12-Month ADV	0.63	0.25	5.87*	Yes
Combined				
1-Month ADV	0.56	0.29	4.36*	Yes
6-Month ADV	0.60	0.29	5.00*	Yes
12-Month ADV	0.61	0.30	5.08*	Yes

Region	Illiquid Return	Illiquid Volatility	Liquid Return	Liquid Volatility	t-Statistic of Long – Short
C. Robustness of illiquidity factor across geographical markets: Returns					
Large					
United States	12.8%	15.7%	9.7%	15.4%	2.43*
United Kingdom (1992–2014)	10.1	18.1	6.8	16.4	1.89
Europe ex-UK	10.4	16.8	9.0	18.9	0.55
Japan (1992–2014)	3.3	17.3	0.3	20.1	1.12
Global	9.5	14.6	8.3	15.7	0.73
Small					
United States	15.8	18.7	11.0	23.9	2.08*
United Kingdom (1992–2014)	5.3	17.4	8.8	19.6	–1.63
Europe ex-UK	11.3	13.8	10.8	20.5	–0.34
Japan (1992–2014)	2.3	18.5	1.7	24.5	–0.30
Global	8.1	12.9	9.4	18.4	–1.16
Combined					
United States	14.4	16.6	10.6	18.9	2.79*
United Kingdom (1992–2014)	7.9	16.5	7.9	17.2	–0.11
Europe ex-UK	11.0	14.5	10.0	19.1	0.04
Japan (1992–2014)	2.9	17.1	1.3	21.2	0.46
Global	8.8	13.2	8.9	16.6	–0.46

(continued)

Table 5. Robustness of the Illiquidity Factor, 1967–2014 (US Data) and 1987–2014 (International Data) (continued)

Region	Sharpe Ratio		t-Statistic of Sharpe Ratio Difference	Significant
	Illiquid	Liquid		
D. Robustness of illiquidity factor across geographical markets: Sharpe ratios				
Large				
United States	0.49	0.30	2.41*	Yes
United Kingdom (1992–2014)	0.41	0.25	1.45	No
Europe ex-UK	0.41	0.29	1.18	No
Japan (1992–2014)	0.03	–0.12	1.38	No
Global	0.42	0.31	1.14	No
Small				
United States	0.57	0.25	5.00*	Yes
United Kingdom (1992–2014)	0.15	0.31	–1.26	No
Europe ex-UK	0.57	0.36	1.63	No
Japan (1992–2014)	–0.02	–0.04	0.16	No
Global	0.36	0.32	0.34	No
Combined				
United States	0.56	0.29	4.34*	Yes
United Kingdom (1992–2014)	0.31	0.30	0.13	No
Europe ex-UK	0.52	0.34	1.86	No
Japan (1992–2014)	0.01	–0.07	0.95	No
Global	0.41	0.33	0.91	No

*Significant at the 5% level.

Sources: Research Affiliates, LLC, using CRSP/Compustat and Worldscope/Datastream data.

- *Distress/financial constraints–related measures:* Dichev (1998) and Piotroski (2000) examined empirical effects of distress.

The industry has about a dozen more ways to define quality, including margins, growth in margins, growth in profitability, financial structure, and earnings stability.[20]

Theoretically, it is hard to argue that high-quality companies should earn a risk premium; labeling these companies "quality" assumes that they are less risky. The most common behavioral argument for why quality companies should earn a higher return is that inattentive market participants fail to incorporate information about company quality into prices.

To test the robustness of the quality factor, we used gross profitability, the popular academic definition (we also used this definition for our international tests), as well as three additional industry definitions of quality: return on equity, gross margins, and leverage. The performance results are reported in **Table 6**. Panels A and B show very few signs of a premium or premium persistence across multiple definitions of quality. Similarly, in the international data in Panels C and D, we see no clear signs of statistical significance.

This apparent lack of robustness may distress readers who have seen many papers (and backtests) that support the existence of a quality premium and its diversification benefit to value investing. We offer a deeper examination of quality investing in Appendix C (posted as supplemental material at www.cfapubs.org/doi/suppl/10.2469/faj.v72.n5.6), in which we describe our tests of many more definitions of quality across three major equity markets and examine their interactions with the value factor.

Downside Risk Characteristics

Risk-averse investors are interested in a more multidimensional view of the potential for underperformance than just the volatility. **Table 7** reports additional downside characteristics of factor portfolios, including return skewness, details of the maximum drawdown event, the longest period of underperformance, and both upside and downside capture.

Panel A of Table 7 shows drawdown characteristics of long-only portfolios. In a long-only setting, skewness values and drawdown events are similar to those of the market. All portfolios had their largest drawdown during either the global financial crisis or the recession of the early 1970s. Most drawdowns were either similar in magnitude or more severe than the drawdowns of the overall market. The exception is low beta, which served its purpose by offering protection to equity investors in those turbulent times.

By netting out market effects, the long–short portfolios in Panel B of Table 7 give us a clearer view of the downside risk characteristics of the factors

Table 6. Robustness of the Quality Factor, 1967–2014 (US Data) and 1987–2014 (International Data)

	Quality		Junk		
Definition	Return	Volatility	Return	Volatility	*t*-Statistic of Long – Short
A. Robustness of quality factor across definitions: Returns					
Large					
Gross profitability	11.1%	16.1%	9.6%	16.5%	0.89
Return on equity	10.5	15.6	10.4	16.9	–0.07
Gross margins	10.2	15.5	10.6	16.2	–0.42
Book leverage	10.6	16.7	9.9	16.2	0.67
Small					
Gross profitability	14.5	22.1	12.5	19.2	2.01*
Return on equity	13.8	20.6	11.1	24.9	1.08
Gross margins	13.2	20.5	13.7	21.6	–0.88
Book leverage	13.3	20.3	13.5	21.3	–0.43
Combined					
Gross profitability	13.0	18.2	11.2	17.0	1.62
Return on equity	12.3	17.3	11.0	20.1	0.65
Gross margins	11.9	17.2	12.3	18.1	–0.76
Book leverage	12.1	17.8	11.8	18.0	0.24

	Sharpe Ratio			
Definition	Quality	Junk	*t*-Statistic of Sharpe Ratio Difference	Significant
B. Robustness of quality factor across definitions: Sharpe ratios				
Large				
Gross profitability	0.37	0.27	1.05	No
Return on equity	0.35	0.31	0.44	No
Gross margins	0.33	0.34	–0.12	No
Book leverage	0.33	0.29	0.50	No
Small				
Gross profitability	0.43	0.39	0.67	No
Return on equity	0.42	0.24	3.22*	Yes
Gross margins	0.40	0.40	–0.05	No
Book leverage	0.41	0.40	0.30	No
Combined				
Gross profitability	0.44	0.36	1.12	No
Return on equity	0.42	0.29	2.22*	Yes
Gross margins	0.40	0.40	–0.10	No
Book leverage	0.40	0.38	0.38	No

	Quality		Junk		
Region	Return	Volatility	Return	Volatility	*t*-Statistic of Long – Short
C. Robustness of quality factor across geographical markets: Returns					
Large					
United States	11.1%	16.1%	9.6%	16.5%	0.89
United Kingdom	10.7	16.8	8.0	20.9	0.75
Europe ex-UK	10.0	17.5	4.9	22.3	1.79
Japan	3.1	20.0	0.6	24.8	0.46
Global	9.8	14.5	6.3	18.3	1.41
Small					
United States	14.5	22.1	12.5	19.2	2.01*
United Kingdom	10.3	20.7	8.2	19.5	1.49
Europe ex-UK	10.9	18.4	9.5	17.9	1.28
Japan	6.2	23.2	4.1	25.9	0.81
Global	12.0	16.7	7.2	16.1	4.05*

(continued)

Table 6. Robustness of the Quality Factor, 1967–2014 (US Data) and 1987–2014 (International Data) (continued)

Region	Quality Return	Quality Volatility	Junk Return	Junk Volatility	*t*-Statistic of Long – Short
Combined					
United States	13.0	18.2	11.2	17.0	1.62
United Kingdom	10.7	17.7	8.3	19.2	1.37
Europe ex-UK	10.6	17.3	7.4	19.4	2.15*
Japan	4.9	20.5	2.6	24.2	0.66
Global	11.0	15.1	6.9	16.3	3.01*

Region	Sharpe Ratio Quality	Sharpe Ratio Junk	*t*-Statistic of Sharpe Ratio Difference	Significant
D. Robustness of quality factor across geographical markets: Sharpe ratios				
Large				
United States	0.37	0.27	1.05	No
United Kingdom	0.43	0.22	1.70	No
Europe ex-UK	0.38	0.07	3.03*	Yes
Japan	–0.02	–0.11	0.73	No
Global	0.44	0.15	2.39*	Yes
Small				
United States	0.43	0.39	0.67	No
United Kingdom	0.33	0.24	1.14	No
Europe ex-UK	0.40	0.34	1.05	No
Japan	0.12	0.02	1.45	No
Global	0.51	0.23	3.64*	Yes
Combined				
United States	0.44	0.36	1.12	No
United Kingdom	0.41	0.25	1.93	No
Europe ex-UK	0.41	0.20	3.06*	Yes
Japan	0.07	–0.03	1.17	No
Global	0.50	0.21	3.40*	Yes

*Significant at the 5% level.

Sources: Research Affiliates, LLC, using CRSP/Compustat and Worldscope/Datastream data.

themselves, which directly translates into out- or underperformance relative to the benchmark for a long-only investor. For example, although low beta outperforms the market during a crash, it still has the potential for extended periods of underperformance. In the raging bull market throughout the 1990s, the long low-beta/short high-beta portfolio lost 76% of its value.

Also notable is the stark contrast in downside characteristics between the value and momentum factors. Momentum is notoriously prone to crashes during swift market reversals. A momentum investor who bet on the continued dominance of high-flying tech stocks in February 2000 and against underperforming (and more reasonably priced) names experienced a 44% decline in less than a year and had to wait more than seven years to get back to even. A value investor experienced a long, drawn-out period of more than a decade of underperformance during the tech boom. Patient value investors, however, were rewarded with a quick recovery—recouping the entire loss in only a year. Momentum investors need to be prepared for a quick and painful loss during a market reversal, whereas value investors need to be prepared for extended periods of slow pain before quick bursts of outperformance. Gains to value strategies, such as the one in 2000, can occur so quickly that investors who wait to see it happening before they get in could miss it entirely.

Panel C of Table 7 shows the upside and downside capture ratios for the long and short sides of each factor. We defined upside (downside) capture as the ratio of portfolio returns to market returns during rising (falling) markets. For low beta, both the upside and the downside capture ratios are below 1; predictably, the low-beta stocks limited both the upside and the downside. With an upside capture ratio of 0.73, returns on the large low-beta portfolio were only 73% of the market return during months when the market went up. When the market went down,

Table 7. Drawdown Characteristics and Downside Capture

Factor	Volatility	Skewness	Maximum Drawdown Event Characteristics: Peak	Trough	Recovery	Return (peak to trough)	Years (peak to trough)	Time to Recover from Trough	Longest Period of Underperformance
A. Drawdown characteristics of the absolute return of the long side of the factor portfolio									
Large cap									
Market	15.3%	–0.44	Oct 07	Feb 09	Feb 12	–49%	1.3	3.0	6.2
Low beta	12.1	–0.19	Nov 07	Feb 09	Apr 11	–37	1.3	2.2	3.4
Value	16.7	–0.29	May 07	Feb 09	Jul 13	–67	1.8	4.4	6.2
Momentum	17.2	–0.45	Oct 07	Feb 09	Sep 12	–49	1.3	3.6	5.1
Illiquidity	15.7	–0.38	May 07	Feb 09	Dec 10	–48	1.8	1.8	3.6
Quality	16.1	–0.30	Dec 72	Sep 74	Jul 80	–52	1.8	5.8	10.8
Small cap									
Market	20.7	–0.39	Nov 68	Dec 74	Jun 77	–60	6.1	2.5	8.6
Low beta	15.0	–0.59	May 07	Feb 09	Feb 11	–50	1.8	2.0	3.8
Value	23.2	0.62	May 07	Feb 09	Apr 10	–71	1.8	1.2	6.5
Momentum	21.2	–0.57	Oct 07	Feb 09	Mar 11	–53	1.3	2.1	3.8
Illiquidity	18.7	–0.10	May 07	Feb 09	Dec 10	–56	1.8	1.8	3.9
Quality	22.1	–0.32	Mar 72	Dec 74	Dec 77	–65	2.8	3.0	5.8
B. Drawdown characteristics of the absolute return of the long–short factor portfolio									
Large cap									
Low beta	16.8%	–0.09	Oct 90	Feb 00	—[a]	–76%	9.3	14.9[a]	24.3[a]
Value	11.8	0.29	Nov 88	Feb 00	Feb 01	–47	11.3	1.0	12.3
Momentum	14.9	–0.52	Feb 00	Jan 01	Jun 08	–44	0.9	7.4	12.3
Illiquidity	10.5	0.65	Apr 88	Feb 00	Aug 04	–57	11.8	4.5	16.3
Quality	10.2	0.03	Jul 72	Dec 06	—[a]	–54	34.4	8.1[a]	42.5[a]
Small cap									
Low beta	16.8	–0.71	Oct 90	Feb 11	—[a]	–70	20.3	3.9[a]	24.3[a]
Value	12.4	0.86	Aug 98	Feb 00	Apr 02	–46	1.5	2.2	5.5
Momentum	16.2	–2.48	Aug 11	Jan 10	—[a]	–71	1.2	5.0[a]	6.2[a]
Illiquidity	10.5	–0.25	Aug 98	Feb 00	Jun 02	–47	1.5	2.3	10.6
Quality	8.3	0.07	May 72	Dec 74	Jan 91	–42	2.6	16.1	18.7
Size	10.9	0.44	Jul 83	Mar 99	Apr 10	–59	15.7	11.1	26.8

Factor	Large: Upside Capture	Large: Downside Capture	Small: Upside Capture	Small: Downside Capture
C. Upside and downside capture of long and short sides of factors				
Low beta				
Low beta	0.73	0.56	0.76	0.59
High beta	1.23	1.40	1.26	1.36
Value				
Value	1.02	0.89	1.04	0.91
Growth	1.02	1.08	1.06	1.15
Momentum				
Winners	1.12	1.02	1.11	0.94
Losers	1.01	1.12	0.98	1.30
Illiquidity				
Illiquid	1.00	0.87	0.91	0.77
Liquid	0.98	1.01	1.11	1.18
Quality				
Quality	1.00	0.97	1.08	1.03
Junk	0.98	1.02	0.90	0.89
Size	1.24	1.18	—	—

[a]Had still not recovered by end of sample.

however, the downside capture of the large low-beta portfolio allowed it to experience only 56% of that loss. Low-beta captures are similar within small caps (0.76 and 0.59), where the market benchmark is the cap-weighted universe of small-cap stocks. This lopsided capture is how low-beta strategies can achieve market-like returns over long periods despite having a beta below 1: They have asymmetrically better performance during the downside, which is an attractive feature for risk-averse investors.

Value, momentum, illiquidity, and quality have upside and downside ratios much closer to 1. Value and illiquidity tend to limit the downside in falling markets and to maintain full capture of the upside in rising markets. Momentum, growth, and quality strategies capture roughly 100% or more of the market's move up or down, with momentum being quite good at winning in rising markets. Size, for which we measured upside and downside captures of small caps vis-à-vis large caps, has equally high upside and downside captures, owing to the higher volatility of the small-cap portfolio. Size, along with growth, has the worst performance in falling markets.

Transaction Costs of Factor Implementation

Paper portfolios studied by academics or presented by index providers rarely include the effect of transaction costs, which is an understandable omission: Studying portfolio returns alone is already important and complex enough. Nonetheless, any investor will attest that transaction costs, as well as fees, are a direct and sizable detractor from portfolio returns. Many of the factors that we examined in our study require fairly concentrated portfolios and frequent rebalancing. For a more realistic understanding of the results of factor investing, we estimated the transaction costs related to various factor definitions.

In this part of our study, we focused on index implementations of factor portfolios. Indexing offers investors several benefits. The transparent, rules-based index construction methodologies make manager monitoring less effortful and, because of strong competition, less expensive. However, transparency and rules-based index construction also bring certain disadvantages. Tight index tracking necessitates inflexible execution: Index replicators have little control over what and when to trade. A pitfall with transparency is that all trades are known to the public ahead of time. This knowledge creates opportunities for front runners to capture a portion of the premium—especially if it is a liquidity-demanding premium—at the expense of index investors.

For the factor portfolios that we considered, as well as for the cap-weighted market portfolio, we used the model developed by Aked and Moroz (2015) to estimate the transaction costs of passively implemented strategies.[21] The price impact reckoned by the model is linearly proportional to the amount of daily liquidity consumed by turnover in a passively traded strategy.[22] Aked and Moroz estimated that the price impact is approximately 30 bps per each 10% of ADV consumed by trading. We used this guideline in calculating the trading costs of various long-only factor portfolios. The fraction of ADV traded depends on the amount invested in each strategy; we assumed US$10 billion in assets under management tracking each of the factor-replicating indexes for large-cap portfolios and US$1 billion for small-cap portfolios.

Note that the *total* amount of assets tracking a particular index is the key determinant of an end investor's transaction costs. If multiple traders rebalance at the same time, their aggregate trading activity determines the overall price impact on investor returns. Investors considering an actively managed strategy are often concerned about its capacity; this concern is also valid with respect to factor investing. The capacity estimates for an index may be defined as the maximum value of total assets managed to the index whereby the factor premium is larger than the incurred costs. Under this definition, the capacity estimates for different factors are inversely proportional to our estimates of the costs.

Despite the evidence that many factors deliver superior performance from their short side, we limited our analysis to the long side because long-only investments are more relevant for the average institutional or private investor. Investors considering long–short factor implementation should be aware that the trading costs for shorting are likely to be significantly higher than our estimates here, which can exacerbate the trading-cost issues for high-turnover strategies.

Do factors remain attractive sources of excess return after adjusting for trading costs? (For our estimates of trading costs, see Appendix D, posted as supplemental material at www.cfapubs.org/doi/suppl/10.2469/faj.v72.n5.6.) **Table 8** shows the long-only portfolios' simulated value-added returns relative to the market, along with their Sharpe ratios, before and after estimated transaction costs. Among the factors we previously found robust, value and low beta largely preserve their advantages even after adjusting for trading costs. Momentum, though quite robust on paper, loses its attractiveness when transaction costs are considered. Similarly, most of the advantages of illiquidity, defined by ADV, disappear when transaction costs are taken into account.

These findings are largely in line with the literature. In a recent study, Novy-Marx and Velikov (2016) reached similar conclusions with a different cost model. They found, as did we, that the transaction costs of liquidity-demanding strategies (including

Table 8. Factors' Added Value and Sharpe Ratios before and after Trading Costs

Factor/Definition	$10 Billion Large-Cap Portfolio: Added Value vs. Market before Transaction Costs	$10 Billion Large-Cap Portfolio: Added Value vs. Market after Transaction Costs	$1 Billion Small-Cap Portfolio: Added Value vs. Market before Transaction Costs	$1 Billion Small-Cap Portfolio: Added Value vs. Market after Transaction Costs
Factor's added value before and after trading costs				
Market-cap weight	0.0%	0.0%	0.0%	–0.1%
Value				
Book to price	2.8%	1.8%	3.9%	1.7%
Earnings to price	3.0	1.6	3.3	0.6
Cash flow to price	2.6	1.7	4.3	1.7
Dividends to price	2.3	1.9	2.7	1.3
Average	2.7%	1.7%	3.6%	1.3%
Momentum				
–2 to –12 Months	2.7%	–3.4%	5.2%	0.4%
–2 to –12 Months, 3-month hold	2.0	–1.6	3.7	0.7
–2 to –12 Months, 1-year hold	0.8	–1.0	2.0	0.6
–2 to –6 Months	0.0	–9.7	2.7	–5.2
–1 to –12 Months	2.1	–3.5	3.8	–0.6
Average	1.5%	–3.8%	3.5%	–0.8%
Low volatility				
Low beta	1.1%	0.1%	2.8%	1.6%
Low volatility	0.4	–0.1	2.6	1.0
Low beta, 3 years	1.1	0.1	2.2	1.3
Low volatility, 3 years	0.7	0.5	2.4	1.2
Average	0.8%	0.2%	2.5%	1.3%
Quality				
Gross profitability	0.8%	0.7%	1.8%	1.4%
Return on equity	0.2	–0.1	1.1	0.2
Gross margins	–0.1	–0.2	0.5	0.1
Book leverage	–0.5	–0.9	0.7	0.1
Average	0.1%	–0.1%	1.0%	0.4%
Illiquidity				
1-Month ADV	2.5%	–2.3%	3.1%	1.8%
6-Month ADV	3.0	–1.1	3.5	2.3
12-Month ADV	2.7	–0.9	3.9	2.8
Average	2.7%	–1.4%	3.5%	2.3%
Size[a]				
50% Small	2.3%	1.8%		
75% Small	2.1	2.1		
25% Small	2.5	–0.1		
Average	2.3%	1.3%		

[a]$10 billion small-cap portfolio.

momentum and a different definition of illiquidity) generally consume all the benefits of such strategies. They also determined, as did we, that the market, value, and low-beta factors remain quite attractive after transaction costs are taken into account.

Yet, these results raise another, more fundamental question: Is full replication of a "factor index" a sound approach to implementing strategies that are based on factors with higher transaction costs? Frazzini, Israel, and Moskowitz (2012) analyzed trading costs associated with the actual implementation of a momentum strategy by an active manager. Their main finding was that, with thoughtful implementation, transaction costs for the momentum strategy can be quite low. Indeed, active managers have a marked advantage over traditional index implementers in this regard. Active managers can be flexible in choosing which securities to trade on the basis of current liquidity conditions. They can be patient in placing their trades. Finally, they can mask their trades to prevent front running. Interestingly, the value that active managers can provide arises not necessarily from their stock-picking skills but, rather, from their ability to actively manage transaction costs in liquidity-taking strategies.

Nevertheless, investors should not discount index-based approaches. Passive implementation usually incurs substantially lower fees than active management but affords the same premiums. Moreover, index designers can use multiple techniques to lower transaction costs.[23] But any reduction in transaction costs from a clever index design may have natural limits in momentum and illiquidity strategies. For factors that can be executed efficiently (i.e., market, value, and low volatility), index implementation seems more advantageous than active management.

Finally, we disregarded taxes in our examination of trading costs. Taxes are generally higher for the higher-turnover strategies. Therefore, tax-adjusted trading costs are probably higher for those strategies with higher estimated trading costs, rendering them even less attractive. As with trading costs, taxes can be reduced with careful execution. Tax-sensitive investors should seek options that are likely to minimize both trading costs and tax liabilities.

Conclusion

We applied heuristic guidelines spelled out in Hsu et al. (2015) to assess the viability of documented return factors: Factors should be grounded in a long and deep academic literature, robust across geographies and definitions, and attractive even after adjusting for transaction costs. We found that most of the factors in the zoo do not have an extensive literature. Lack of academic interest, as revealed by the low number of published papers, signals one of two things: (1) The original results have low reliability or replicability, or (2) the documented phenomenon fits such a narrow niche that it holds little interest for a broader readership.

We identified six factors, summarized in **Table 9**, with a deep literature: illiquidity, low beta, value, momentum, size, and quality. To assess the persistence and reliability of these factors, we conducted tests to determine whether they would stand up to small perturbations in definitions and deliver premiums in various countries and regions. We found that two of the more popular factors—quality and size—lack robust empirical evidence to support them.

We also studied the implementation of factor-based strategies and concluded that factors naturally form two groups: more liquidity demanding, which includes momentum and illiquidity, and less liquidity demanding, which includes value and low beta. Active managers who are skilled at execution and have attractive fee schedules may be better suited to execute strategies that target the more liquidity-demanding factors. A well-constructed index can deliver to end investors most of the benefits of the value and low-beta factors at low cost.

Many of the issues concerning factor investing lie beyond the scope of this study. One of the more important is how to combine factors into a portfolio. The simplest approach to factor allocation can be to select factors that the investor determines to be robust and assign equal weights to them. This approach should provide diversified exposure to factors and, because of its simplicity, should be easy from a governance perspective. When selecting factors and deciding on their relative weights in the portfolio, investors who go beyond equal weighting should consider their correlations. Favoring negatively correlated factors improves the portfolio's risk–return characteristics—for example, value and momentum characteristics are often negatively correlated. That does not mean, however, that negative correlations mitigate concerns about trading costs. Investors who wish to combine momentum and value should seek vehicles that will preserve the factor premium after the execution costs.

Another issue, mentioned earlier, is factor capacity. If factor investing continues to attract assets at the current rate, some factor-based strategies—probably starting with the high-trading-cost strategies—may become crowded. Perhaps investment proportional to factor capacity (which is, as we pointed out, inversely related to trading costs) will be more sustainable.

Finally, factor premiums may not be constant over time. As with any strategy, there is considerable randomness around the average outcome. (Appendix E, posted as supplemental material at www.cfapubs.org/doi/suppl/10.2469/faj.v72.n5.6, provides estimates of factor performance in subperiods of 1967–2014.) Cohen, Polk, and Vuolteenaho (2003); Lou and Polk (2013); and Garcia-Feijóo, Kochard, Sullivan, and Wang (2015) have demonstrated that the value, momentum, and low-beta factor premiums, respectively, are time varying: They can be crowded and expensive at some times and cheap at other times.[24] In addition, factors' risk profiles can vary over time. For example, the low-beta factor may be somewhat crowded at present relative to its historical levels because low-beta strategies have become widely accepted as a legitimate investment option in the last 10–15 years. The result may be disappointing returns in the near future.

Assessing factor valuations from a historical perspective can help sophisticated investors make timing decisions. We caution, however, that tactical allocations generally cannot diversify the breadth of exposure and thus tend to have low information ratios. Perhaps these studies' findings are more

Table 9. Summary of Findings

		Robustness across Definitions (no. of statistically significant outcomes vs. no. of variations tested)		Robustness across Geographical Markets (no. of statistically significant outcomes vs. no. of variations tested)			Annual Premium ($10 billion large-cap portfolio)			Annual Premium ($1 billion small-cap portfolio)		
Factor	No. of SSRN Search Outcomes	Information Ratio Tests	Sharpe Ratio Tests	Information Ratio Tests	Sharpe Ratio Tests	Robust across Definitions and Markets	Before Trading Costs	After Trading Costs	Added Value Survives Costs	Before Trading Costs	After Trading Costs	Added Value Survives Costs
Low beta	260	0 out of 12	12 out of 12	0 out of 15	13 out of 15	Yes, Sharpe ratio only	1.1%	0.1%	No	2.8%	1.6%	Yes
Value	2,327	8 out of 12	12 out of 12	13 out of 15	13 out of 15	Yes	2.8	1.8	Yes	3.9	1.7	Yes
Momentum	457	8 out of 15	11 out of 15	9 out of 15	12 out of 15	Yes	2.7	–3.4	No	5.2	0.4	No
Illiquidity	570	9 out of 9	9 out of 9	3 out of 15	3 out of 15	Mixed evidence	2.5	–2.3	No	3.1	1.8	Yes
Quality	1,700	1 out of 12	2 out of 12	4 out of 15	5 out of 15	No	0.8	0.7	No	1.8	1.4	Yes
Size	1,167	2 out of 3	0 out of 3	1 out of 5	0 out of 5	No	2.3	1.8	Yes			

Sources: Research Affiliates, LLC, using CRSP/Compustat and Worldscope/Datastream data.

useful for pointing out what *not* to do: Elton, Gruber, and Blake (2006) and Goyal and Wahal (2008) documented for 401(k) plans and plan sponsors, respectively, that investors are prone to engage in trend chasing. They are disposed to fire managers after a period of underperformance and hire new managers who have recently outperformed. This behavior tends to hurt their portfolios' performance. If investors similarly rotate between factors on the basis of recent performance, they run the risk of disinvesting when they should invest and vice versa. Investors who choose factor-based strategies will benefit from a disciplined buy-and-hold policy that resolutely disregards short- and medium-term performance.

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Appendix A. Factor Definitions

Within each category, the first listed definition is the most common. These are also the definitions used for international tests.

Signal	Definition
Value	
Book to price	Book/Market cap
Earnings to price	Trailing earnings/Market cap
Cash flow to price	Cash flow/Market cap
Dividends to price	Dividends/Market cap
Momentum	
–2 to –12 Months, 1-month hold	Prior 12 months' returns, skipping most recent month, hold for one month (monthly balance)
–2 to –12 Months, 3-month hold	Same as above, hold for three months (quarterly rebalance)
–2 to –12 Months, 1-year hold	Same as above, hold for one year (annual rebalance)
–2 to –6 Months	Prior six months' returns, skipping most recent month
–1 to –12 Months	Prior 12 months' returns
Low beta	
Low beta	Frazzini and Pedersen definition $\beta_i = \rho(\sigma_i/\sigma_m)$, where ρ is estimated with five years of daily local currency returns and σ with one year of daily local currency returns
Low volatility	Volatility over prior year using daily return data
Low beta, 3 years	Market beta over prior three years using daily return data
Low volatility, 3 years	Volatility over prior three years using daily return data
Quality	
Gross profitability	(Revenue – Cost of goods sold)/Assets
Return on equity	Net income/Book
Gross margins	(Revenue – Cost of goods sold)/Sales
Book leverage	Debt/Book
Illiquidity	
1-Month ADV	Average daily volume of prior month
6-Month ADV	Average daily volume of prior six months
12-Month ADV	Average daily volume of prior 12 months
Size	
50% Small, 50% big	Size breakpoint = NYSE median market cap
75% Small, 25% big	Size breakpoint = NYSE 75th percentile market cap
25% Small, 75% big	Size breakpoint = NYSE 25th percentile market cap

Notes

1. Harvey, Liu, and Zhu (2016) found 316 factors in the literature as of year-end 2014 and also found that approximately 40 new factors are published annually. These two facts suggest that as of year-end 2016, we will be at 300 + 80 or higher.
2. John Cochrane coined the term "zoo of new factors" in his presidential address at an annual meeting of the American Finance Association (Cochrane 2011).
3. We limited the scope of our study to examining which factors can profit investors on a standalone basis. If we found that a certain factor lacks robustness, such a finding would not imply that this factor might not be important in the broader asset-pricing context (e.g., owing to its correlations with other factors).
4. Indexing is often associated with capitalization-weighted benchmarks. Smart beta, which breaks the link between asset prices and index weights, is another approach to index investing; it is designed to capture nonmarket sources of premiums.
5. We searched for keywords that included a factor name combined with the word *factor* and required at least 100 hits for a factor to be included in our study. Our rationale for using the word *factor* in each query was to home in on asset-pricing papers rather than, say, corporate finance papers.
6. The difference between our approach and that of Harvey et al. (2016) is that we further aggregated several of the distinct factors into groups that are more common among practitioners and that can be logically combined. For instance, we classified several ratios (e.g., P/E, P/B, and P/D) as value factors.
7. This phenomenon is often referred to as a flat or even inverted security market line (SML), which is often found in empirical studies. A flat SML means that average stock performance is largely unrelated to the riskiness of the stocks; an inverted SML means that low-risk stocks tend to slightly outperform riskier stocks.
8. For references, see Black, Jensen, and Scholes (1972) and Frazzini and Pedersen (2014).
9. For evidence of investors' preference for gambling and the low-beta anomaly, see Blau, Hsu, and Whitby (2014); Bali, Brown, Murray, and Tang (2015); and Hsu and Viswanathan (2015). For evidence that investors in emerging markets use the stock market as a gambling substitute, see Gao and Lin (2015). For another reading of the preference-for-gambling hypothesis, see Baker, Bradley, and Wurgler (2011).
10. See Hsu, Kudoh, and Yamada (2013).
11. See Baker, Bradley, and Wurgler (2011) and Brennan, Cheng, and Li (2012).
12. For factor definitions, see Appendix A.
13. Frazzini and Pedersen (2014) constructed a long–short factor portfolio labeled "betting against beta" (BAB), which levers up the low-beta stocks (by about 1.4× for US stocks) on the long side and shorts the high-beta stocks (by only 0.7×), so this portfolio has a beta of approximately zero with respect to the equity market.
14. MSCI estimates that roughly $50 billion is tied to its minimum-volatility index, which is dwarfed by the estimated $7 trillion of assets tied to various cap-weighted market indexes.
15. For a full theoretical treatment of overreaction and momentum, see Hong and Stein (1999).
16. For an analysis of momentum transaction costs, see Grundy and Martin (2001).
17. See De Bondt and Thaler (1985).
18. We followed the definition in Fama and French (2012).
19. For other, more complicated definitions of illiquidity, see Amihud (2002) and Pástor and Stambaugh (2003).
20. For a more exhaustive list of ways to define quality, see Appendix C (posted as supplemental material at www.cfapubs.org/doi/suppl/10.2469/faj.v72.n5.6).
21. In this analysis, we focused on US strategies for the sake of brevity. Because this trading-cost model concerns ADV characteristics of turnover, we saw similar patterns in trading costs in international markets where the behavior of these attributes is similar.
22. Although the linear model we chose is very simple, it is extremely easy to estimate for historical datasets because it does not require additional historical data beyond ADV. Moreover, despite its simplicity, it provides realistic estimates for trades that do not consume too much liquidity. Like Aked and Moroz (2015), Vangelisti (2006) estimated an impact of 30 bps per 10% of ADV. But the linear price impact model has limitations when trading volume is large. Gabaix, Gopikrishnan, Plerou, and Stanley (2006) found that for larger trades (equivalent to 200% or more of ADV), the price impact grows at a lower-than-linear rate (approximately the square root of trading volume). The slower-than-linear impact found by Gabaix et al. implies that the Aked–Moroz methodology we used overestimates trading costs for extremely high-turnover strategies. Nonetheless, in our study, the assumed level of assets under management ensured that most trades stayed below the 200% ADV level, where the nonlinearity effects start to matter.
23. These techniques include (1) designing indexes with high weighted-average market caps, (2) eliminating unnecessary turnover, (3) placing bands or tolerance zones around index boundaries to reduce the expensive turnover that might otherwise arise from stocks jumping in and out of the index, and (4) spreading turnover with staggered rebalancing by using a methodology similar to the one introduced by Blitz, van der Grient, and van Vliet (2010). Such techniques can reduce transaction costs appreciably. In our simulation, we used simplified methods for constructing factor portfolios.
24. There is a long literature on market factor timing, including Campbell and Shiller (1988); Welch and Goyal (2008); and Cochrane (2008). Other noteworthy studies that document the variability of factor premiums over time include Asness, Friedman, Krail, and Liew (2000; value); Daniel and Moskowitz (2013; momentum); and Li and Lawton (2014; low beta).

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